Supplementary material

The nonlinear viscoelastic (VE) strain was based on Schapery's non-linear viscoelastic constitutive law,[1] given by Manda et al.[2]

$$\varepsilon_{ve}(t) = g_0 D_g \sigma + \int_0^t \Delta D\left(\psi^t - \psi^\tau\right) \frac{d(g_2\sigma)}{d\tau} d\tau \qquad (1)$$

$$\psi^t = \int_o^t \frac{d\tau'}{\alpha_{\sigma(\tau')}\alpha_{T(\tau')}\alpha_{e(\tau')}} \qquad (2)$$

where D_g is instantaneous compliance, g_0, g_1, g_2 and α_σ are stress-dependent non-linear viscoelastic parameters expressed as second order polynomial equations,[2] σ is applied stress and ψ^t is reduced time. The effects of temperature (α_T) and other environmental variables (α_e) are not considered, consequently these two parameters are unity. The transient compliance, ΔD, in equation (1) is represented by Prony series as

$$\Delta D\left(\psi^t\right) = \sum_1^n D_n\left[1 - \exp\left(-\lambda_n \psi^t\right)\right] \qquad (3)$$

where D_n is nth coefficient of the Prony series associated with the reciprocal of nth retardation time, λ_n.

Manda et al[2] used this model to evaluate parameters for 19 trabecular bone samples; three samples, with BV/TV of 15%, 25% and 35%, were selected from this study. The corresponding parameters and their definitions are provided in Table i.

The study by Manda et al[2] found that trabecular bone response to mechanical loads comprises both recoverable and irrecoverable deformations. Our nonlinear viscoelastic-viscoplastic (VEP) model incorporated these, from the data available from the above study, by employing a viscoplastic constitutive model along with the non-linear viscoelastic model. In this, the total strain rate is given by

$$\dot{\varepsilon}_{total} = \dot{\varepsilon}_{ve} + \dot{\varepsilon}_{vp} \qquad (4)$$

where $\dot{\varepsilon}_{ve}$ and $\dot{\varepsilon}_{vp}$ are viscoelastic and viscoplastic strain rates, respectively. The viscoplastic strain rate based on Perzyna model,[3] is given by

$$\dot{\varepsilon}_{vp} = \eta \left\langle \left(\frac{F}{\sigma_y^0}\right)^N \right\rangle \frac{\partial G}{\partial \sigma} \qquad (5)$$

Table i. The nonlinear viscoelastic parameters along with linear Prony coefficients and irrecoverable strains at multiple stress levels for three different BV/TV [1]

BV/TV	Linear coefficients at Cycle I	Cycle n	ε_{static} (%)	σ^N	Nonlinear VE parameters				ε_{irrec} (%)
					g_0	g_1	g_2	α_{sigma}	
15%	$\begin{bmatrix} D_g \\ D_1 \\ D_2 \\ D_3 \\ \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 6.40\times10^{-3} \\ 5.48\times10^{-4} \\ 3.24\times10^{-4} \\ 2.97\times10^{-4} \\ 8.64\times10^{-3} \\ 8.64\times10^{-1} \\ 9.31\times10^{-2} \end{bmatrix}$	I	0.2	0.36	1.00	1.00	1.00	1.00	0.041
		II	0.4	0.66	0.91	1.06	0.59	0.78	0.067
		III	0.6	0.94	0.94	1.03	0.67	0.82	0.104
		IV	0.8	1.17	0.99	1.01	0.82	0.85	0.158
		V	1.0	1.35	1.10	0.96	0.84	0.91	0.237
25%	$\begin{bmatrix} D_g \\ D_1 \\ D_2 \\ D_3 \\ \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 3.52\times10^{-3} \\ 1.31\times10^{-4} \\ 2.63\times10^{-4} \\ 1.30\times10^{-4} \\ 7.57\times10^{-2} \\ 6.44\times10^{-3} \\ 5.68\times10^{-1} \end{bmatrix}$	I	0.2	0.64	1.00	1.00	1.00	1.00	0.032
		II	0.4	1.20	0.90	1.02	0.82	0.79	0.049
		III	0.6	1.77	0.91	1.05	0.96	0.75	0.084
		IV	0.8	2.23	0.98	1.04	1.19	0.74	0.140
		V	1.0	2.43	1.06	1.01	1.44	0.81	0.209
35%	$\begin{bmatrix} D_g \\ D_1 \\ D_2 \\ D_3 \\ \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 1.60\times10^{-3} \\ 1.14\times10^{-4} \\ 6.45\times10^{-5} \\ 8.35\times10^{-5} \\ 7.64\times10^{-3} \\ 9.41\times10^{-2} \\ 7.05\times10^{-1} \end{bmatrix}$	I	0.2	1.31	1.00	1.00	1.00	1.00	0.039
		II	0.4	2.69	0.84	1.14	0.71	0.67	0.057
		III	0.6	4.09	0.84	1.08	0.60	0.78	0.072
		IV	0.8	5.59	0.82	1.00	0.57	0.87	0.075
		V	1.0	7.50	0.78	1.00	0.37	0.93	0.109
		VI	1.5	13.01	0.70	1.02	0.66	0.80	0.214

BV/TV is the bone volume fraction, D_g is the instantaneous compliance in 1/MPa, D_n (n = 1, 2, 3) are transient compliance coefficients in 1/MPa, λ_n (n = 1, 2, 3) are reciprocal of nth retardation time in Prony series in s^{-1}, ε_{static} is the applied static strain in each loading cycle, σ^N is the stress corresponding to plateau stress in the Nth loading cycle in MPa. Parameters g_0, g_1, g_2, and α_σ are stress-dependent nonlinear viscoelastic (VE) parameters and ε_{irrec} is the irrecoverable strain exist at the end of each loading cycle.

Table ii. The values of the viscoplastic parameters for three different BV/TV

BV/TV	α	β	N	$\eta\left(s^{-1}\right)$	κ_0	κ_1	κ_2	σ_y^0
15%	1.035	0	3	4.34×10^{-2}	2.42×10^{-3}	2.11	4.62×10^{2}	3.38
25%	1.035	0	3	4.91×10^{-3}	1.35×10^{-10}	4.11	3.50×10^{2}	5.75
35%	1.035	0	3	5.35×10^{-2}	4.03×10^{-1}	10.40	5.00×10^{2}	26.70

BV/TV is the bone volume fraction, α is a pressure-sensitivity parameter related to friction angle, β is a parameter related to dilation angle, σ_y^0 and N are material constant, η is a viscosity parameter, κ_0 is the initial yield stress, κ_1 is the saturated stress for the fully hardened material, κ_2 is the transition rate between κ_0 and $\kappa_0+\kappa_1$.

where η is the viscoplastic parameter, G is viscoplastic potential function. The Macaulay brackets, <·>, indicate that the viscoplastic strain rate is non-zero only when $\left(\frac{F}{\sigma_y^0}\right)^N > 0$. The terms σ_y^o and N are material parameters. We based the yield function F on extended Drucker-Prager criterion

$$F = \tau - \alpha p - \kappa\left(\varepsilon_{vp}\right) \quad (6)$$

where τ is deviator shear stress; $p = -\frac{1}{3}tr(\sigma)$ is the equivalent pressure stress; $\alpha = \tan\theta$ is a pressure-sensitivity parameter related to friction angle θ; $\kappa\left(\varepsilon_{vp}\right)$ is viscoplastic hardening function, which is a function of effective viscoplastic strain, given by

$$\kappa\left(\varepsilon_{vp}\right) = \kappa_0 + \kappa_1\left[1 - \exp\left(-\kappa_2\varepsilon_{vp}\right)\right] \quad (7)$$

where κ_0 is the initial yield stress, κ_1 is the saturated stress for a fully hardened material, κ_2 is the transition rate between κ_0 and $\kappa_0+\kappa_1$. The viscoplastic potential function is given by

$$G = \tau - \beta p \quad (8)$$

where $\beta = \tan\theta'$ is a parameter related to the dilation angle θ'. In this study, the friction angle θ and dilation angle θ' were assumed to be 46^o and 0^o, respectively, based on a previous investigation on bone.[4] Consequently, α and β were equal to 1.035 and 0, respectively.

The evaluated viscoplastic parameters for three BV/TV bone samples considered in this study are given in Table ii. These parameters were used as input to the user defined material (UMAT) for trabecular bone samples, which was implemented in Abaqus 6.12 (Simulia, Providence, Rhode Island).

References

1. **Schapery RA.** On the characterization of nonlinear viscoelastic materials. *Polym Eng Sci* 1969;9:295–310.
2. **Manda K, Wallace RJ, Xie S, Levrero-Florencio F, Pankaj P.** Nonlinear viscoelastic characterization of bovine trabecular bone. *Biomech Model Mechanobiol* 2016;16:173–89.
3. **Perzyna P.** Thermodynamic Theory of Viscoplasticity. *Adv Appl Mech* 1971;11:313–354.
4. **Mercer C, He MY, Wang R, Evans AG.** Mechanisms governing the inelastic deformation of cortical bone and application to trabecular bone. *Acta Biomater* 2006;2:59–68.